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COMPARISON OF ANALYTICAL AND FINITE ELEMENT RESULTS FOR DEFLECTIONS OF CDF YOKE AND ENDPLUG

by

R. Wands

Novmeber 18, 1982

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The purpose of this report is to compare the deflection results obtained by the finite element analysis of the CDF yoke and endplug with results arrived at by conventional analytical means. The analyzed structures are shown in Figs. 1 and 2. The source for the closed form solutions is "Formulas for Stress and Strain", by Raymond Roark and Warren Young. The tabulated comparisons of four deflections are given in Table I. Following the table are the details of the calculations.

Table I
Comparison of Analytical and Finite Element Deflection
for CDF Yoke and Endplug

Structure and Loading	Calculation	Deflections: Analytical	Deflections: Finite Element
Axial deflection of yoke under axial electromagnetic load	1	-.03	-.11
	2	-.30	
	3	-.014	
	4	-.075	
	5	-.025	
Vertical deflection at midspan of lower return leg under roman arch loading	6	-.018	-.013
Vertical deflection at midspan of upper return leg under its own weight	7	-.005	-.007
Axial deflection of endplug under axial electromagnetic load	8a	.121	.038
	8b	.037	
	8c	.0167	
	8d	.008	

A. Axial Deflection of Yoke Under Axial Electromagnetic Load (Fig. 3)

Calculation #1

Assumptions:

1. Yoke endwall is modeled as a circular plate 212" in radius, 36" thick, with a circular hole at its center of 60" radius.

2. Yoke endwall is simply supported at its outer circumference.

3. Inner edge of 60" radius hole is free.

Using formula for case 1a, pp 334 Roark and Young,

$r_o = b = 60''$

$a = 212''$

$t = 36''$

$w = 1.4\ (10^6)\ \text{lbs}/(2\pi\ .60) = 3714\ \text{lbs/in}$

$D = 1.28\ (10'')$

$K_y = -.12$

$$y = \frac{K_y\ wa^3}{D} = \frac{-.12\ (3714)\ 212^3}{1.28\ (10'')} = .03''$$

Calculation #2

Assumptions:

1. Yoke endwall is modeled as circular plate 212" in radius with a circular hole at its center of 60" radius.

2. The thickness is equivalent to an area weighted average over the 36" thick region and the 2" thick region.

3. Yoke endwall is simply supported at its outer circumference.

4. Inner edge of 60" radius hole is free.

5. Stiffening effect of endwall ribs is neglected.

Using formula for case 1a, pp 334 of Roark and Young,

$r_o = b = 60''$

$a = 212''$

$t = 17''$

$w = 3714$ lbs/in

$D = 1.4\ (10^{10})$

$K_y = -.12$

$$y = \frac{K_y\ wa^3}{D} = \frac{-.12\ (3714)\ 212^3}{1.4\ (10^{10})} = -.30''$$

Calculation #3

Assumptions:

1. Yoke endwall is modeled as a circular plate 150" in radius, with a circular hole at its center of 60" radius.
2. The thickness is equivalent to an area weighted average over the 36" thick region and the 2" thick region.
3. Yoke endwall is simply supported along its outer circumference.
4. Inner edge of 60" radius hole is free.
5. Stiffening effect of endwall ribs is neglected.

Using formula for case 1a, pp 334 of Roark and Young

$r_o = b = 60''$

$a = 150''$

$t = 17''$

$w = 3714$ lbs/in

$D = 1.4\ (10^{10})$

$K_y = -.16$

$$y = \frac{K_y\ wa^3}{D} = \frac{-.16\ (3714)\ 150^3}{1.4\ (10^{10})} = .14''$$

Calculation #4

Assumptions:

1. Yoke endwall is modeled as circular plate 212" in radius, with a circular hole at its center of 60" radius.

2. The thickness is equivalent to an area weighted average over the 36" thick region and the 2" thick region.

3. Yoke endwall is fixed at its outer circumference.

4. Inner edge of 60" radius hole is free.

5. Stiffening effect of endwall ribs is neglected.

Using formula for case 1e, pp 336 of Roark and Young

$r_o = b = 60"$

$a = 212"$

$t = 17"$

$w = 3714$ lbs/in

$D = 1.4\ (10^{10})$

$K_y = -.031$

$$y = \frac{K_y\ wa^3}{D} = \frac{-.031\ (3714)\ 212^3}{1.4\ (10^{10})} = -.078$$

Calculation #5

Assumptions:

1. Yoke endwall is modeled as a circular plate 150" in radius, with a circular hole at its center of 60" radius.

2. The thickness is equivalent to an area weighted average over the 36" thick region and the 2" thick region.

3. Yoke endwall is fixed at its outer circumference.

4. Inner edge of 60" radius hole is free.

5. Stiffening effect of endwall ribs is neglected.

Using formula for case 1e, pp 336 of Roark and Young

$r_o = b = 60"$

$a = 212"$

$t = 17"$

$w = 3714$ lbs/in

$D = 1.4\ (10^{10})$

$K_y = -.028$

$$y = \frac{K_y\ wa^3}{D} = \frac{-.028\ (3714)\ 150^3}{1.4\ (10^{10})} = .025"$$

B. <u>Vertical Deflection at Midspan of Lower Return Under "Roman Arch" Loading (Fig. 4)</u>

<u>Calculation #6</u>

<u>Assumptions:</u>

1. Return leg is modeled as a simply supported beam with concentrated loads.

Using formula from p 96 of Roark and Young

$$y = y_A + O_A X + \frac{M_A X^2}{2EI} + \frac{R_A X^3}{6EI} - \frac{P}{6EI}(x - a)^3$$

and formula for case 1e, pp 97 of Roark and Young, with

$$EI = 30(10^6)\text{psi}\ \frac{1}{12}\ (112)\ 24^3 = 3.87\ (10^{12})\ \text{lb/in}^2$$

$\ell = 204$ in

$x = 102$ in

then for $P_1 = 90\ (10^3)$ lbs,

a) $y_A = 0$

b) $$O_A X = \frac{-P_1\ a_1\ (2\ell - a_1)(\ell - a_1)x}{6EI\ell} = \frac{-90(10^3)(2 \cdot 204 - 11)(204-11)(102)}{6(204)(3.87)10^{12}}$$

$$= -.002 \text{ in}$$

c) $$\frac{M_A X^2}{2EI} = 0 \quad (M_A = 0)$$

d) $$\frac{R_A X^3}{6EI} = \frac{P_1 \ (\ell - a_1) x^3}{\ell \ 6EI} = \frac{90(10^3)(204 - 11)102^3}{204(6)3.87(10^{12})}$$

$$= .004 \text{ in}$$

e) $$\frac{-P}{6EI} \langle x - a_1 \rangle = \frac{90(10^3)(102 - 11)^3}{6(3.87)10^{12}} = -.003 \text{ in}$$

Then, total y deflection due to P_1 is

$$y_{tot} = -.002 + .004 - .003 \quad = -.002 \text{ in}$$

For $P_2 = 90(10^3)$

a) $y_A = 0$

b) $$0_A x = \frac{P_2 a_2 (2\ell - a_2)(\ell - a_2) x}{6EI\ell} = \frac{-90(10^3)(91)(2(204-91)(204 - 91)102}{6(204)(3.87)10^{12}}$$

$$= -.006$$

c) $$\frac{M_A x^2}{2EI} = 0 \quad (M_A = 0)$$

d) $$\frac{R_A x^3}{6EI} = \frac{P_2 (\ell - a_2) x^3}{6EI\ell} = \frac{90(10^3)(204 - 91)(102)^3}{204(6)(3.87)10^{12}}$$

$$= .002$$

e) $$\frac{-P}{6EI} \langle x - a_2 \rangle^3 = \frac{-90(10^3)(102 - 91)^3}{6(3.87)10^{12}} = 7(10^{-6}) \text{ negligible}$$

Then, total deflection due to P_2 is

$$y_{tot} = 2 \quad -.006 + .002 \quad = -.008 \text{ in}$$

For $P_3 = 90(10^3)$, (loading is at midspan and calculation is simplified)

$$y_{max} = 2\frac{-P_3\,\ell^3}{48EI} = 2\frac{-90(10^3)(204)^3}{48(3.87)10^{12}}$$

$$= -.008 \text{ in}$$

The total deflection occuring at the midspan of the return leg is

$$Y = -.002 - .008 - .008 = -.018 \text{ in.}$$

C. Vertical Deflection at Midspan of Upper Return Leg Under its Own Weight (Fig. 5)

Calculation #7

Assumptions:

1. Deflection of endwall under weight of leg is sum of bending in leg and deflection of vertical endwall members.

2. Model as a simply supported beam of uniform cross section.

Using formula for case 2e, pp 100 of Roark and Young then

$EI = 3.87\ (10^{12})\ \text{lb/in}^2$

$w = 763$ lbs/in

$\ell = 204$ in

$$y_{max} = \frac{-5w\ell^4}{384\ EI} = \frac{-5(763)(204)^4}{384(3.87)10^{12}}$$

$$= -.004 \text{ in}$$

The deflection of the endwall under the weight of the return leg is found by calculating the stiffness of the vertical endwall members and dividing by the weight on the endwall.

$$K = \frac{AE}{\ell}$$

where

K = stiffness of endwall

A = total area of two endwall members

E = Young's modulus

ℓ = vertical height of endwall

$$K = \frac{2(8)36(30)10^6}{300} = 5.75\ (10^7)\ \text{in}$$

Total load on the endwall is one half of total return leg weight.

$$F = \frac{763\ (204)}{2} = 77826$$

then

$$y = \frac{77826}{5.76(10^7)} = -.001$$

Then the total y deflection is

$$y_{tot} = -.004 - .001 = -.005 \text{ in}$$

D. <u>Axial Deflection of Endplug Under Axial Electromagnetic Load (Fig. 6)</u>

<u>Calculation #8</u>

<u>Assumptions:</u>

1. Correlations for circular plates with holes are applicable.

2. The ribs and straps connecting the twenty endplug plates serve to keep the plates separated by some constant amount.

3. The dominating displacement is due to plate bending.

4. The electromagnetic pressure forces can be converted to equivalent anular concentrated loads which can be applied to a spring whose stiffness is the sum of the twenty plate bending stiffnesses. This is the endplug analytical model of Fig. 6.

5. The ribs in plates 5 through 20 serve only to decrease the effective outside diameter for stiffness calculation.

Equations for deflection due to pressure and deflection due to concentrated annular loads were obtained from case 2a and case 1a on pp 339 and 334 respectively of Roark and Young. Equating these deflections allows calculation of a concentrated annular load which produces the same displacements on an isolated plate as the electromagnetic pressure force. The bending stiffness of all plates is calculated and summed, as are all equivalent anular loads. Then,

$$y_{max} = F/K$$

where

F = concentrated annular load equivalent to pressure loadings

K = total bending stiffness of plates

This calculation was performed with a short computer program which did the stiffness and force calculations. Four different boundary conditions were considered:

Calculation	Outer Circumference	Inner Circumference	Y_{max}
8a	simply supported	free	.121 in
8b	simply supported	guided	.037
8c	fixed	free	.0167
8d	fixed	guided	.008

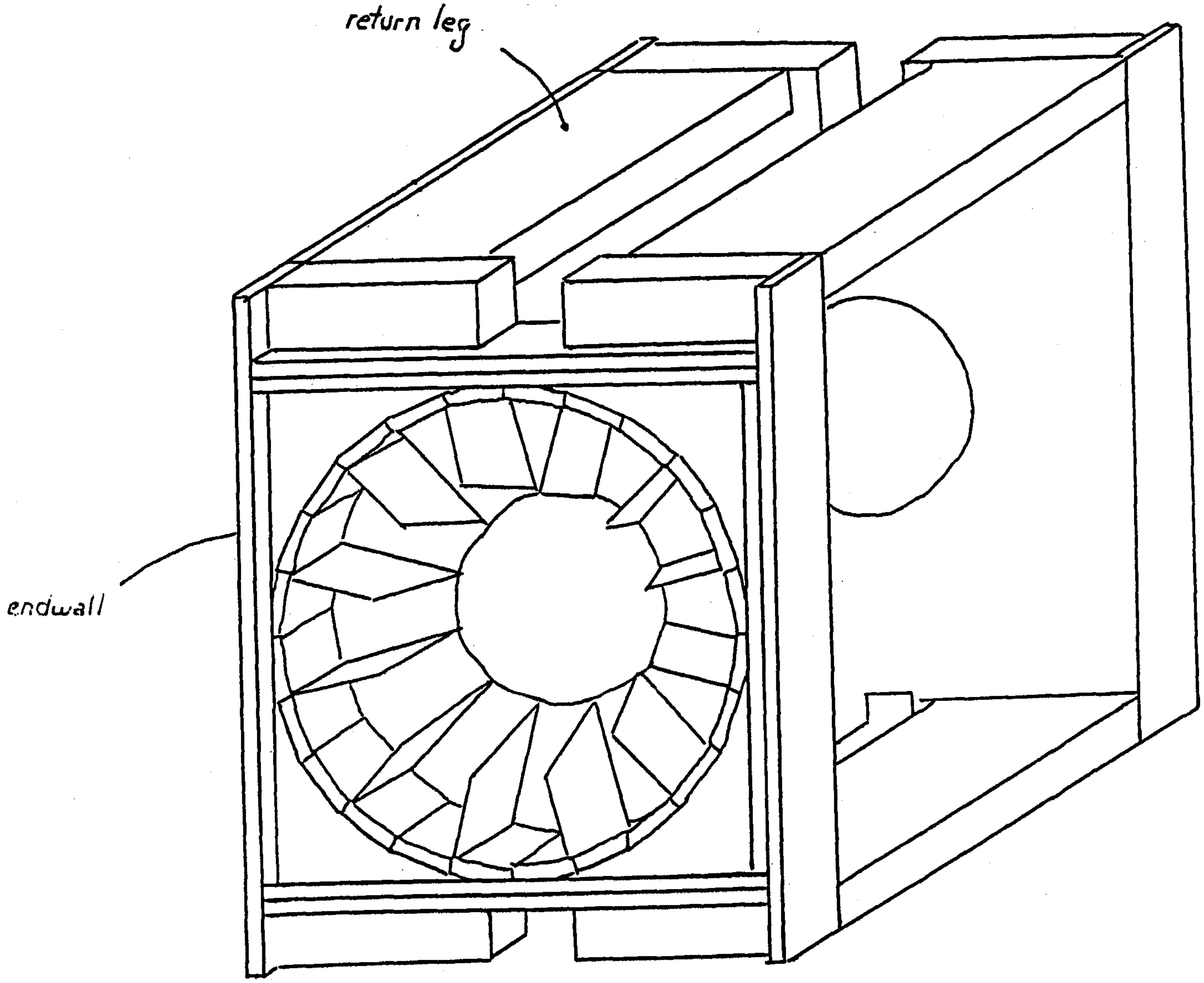


Fig. 1. CDF Yoke

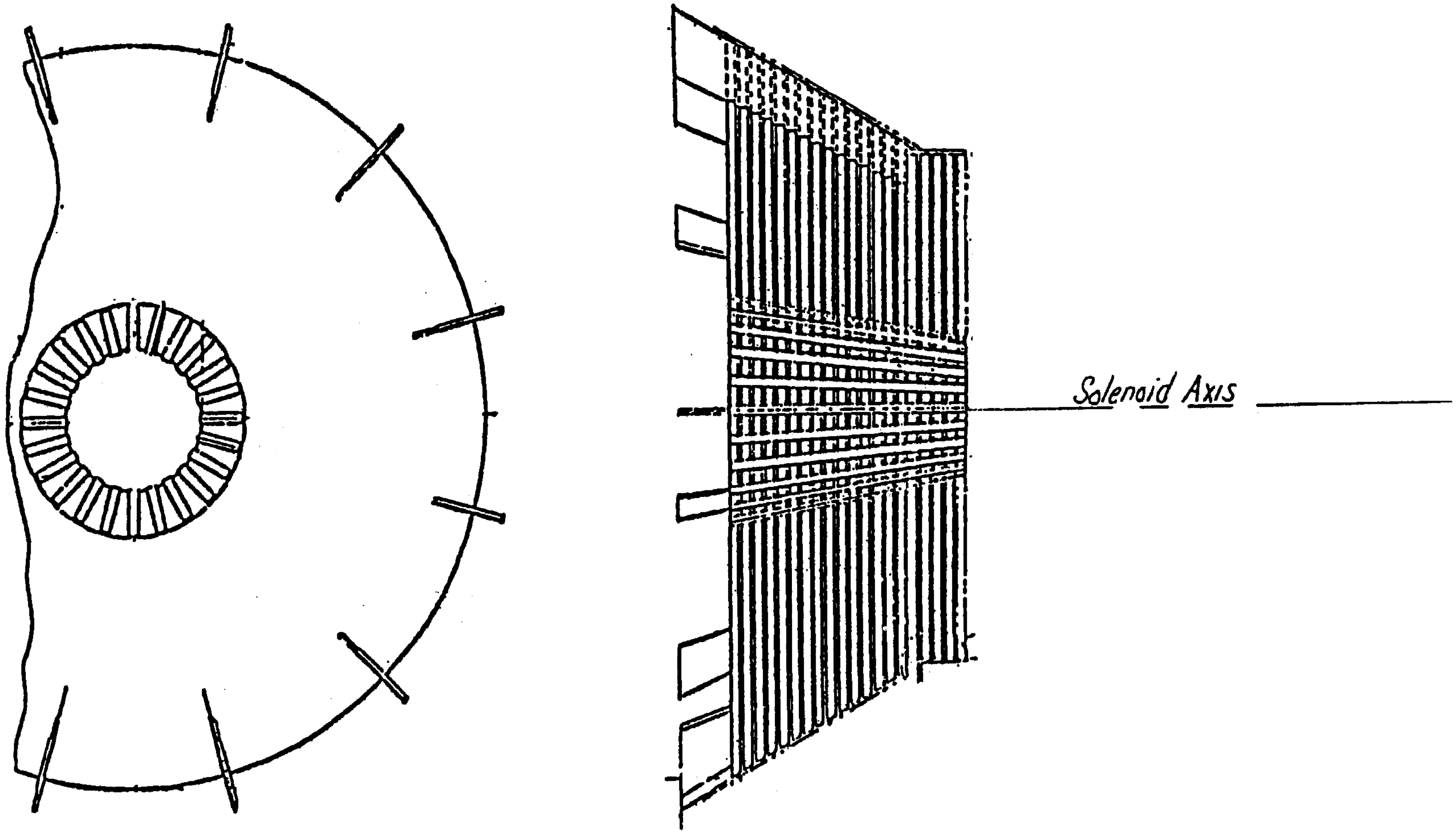


Fig 2. CDF Endplug

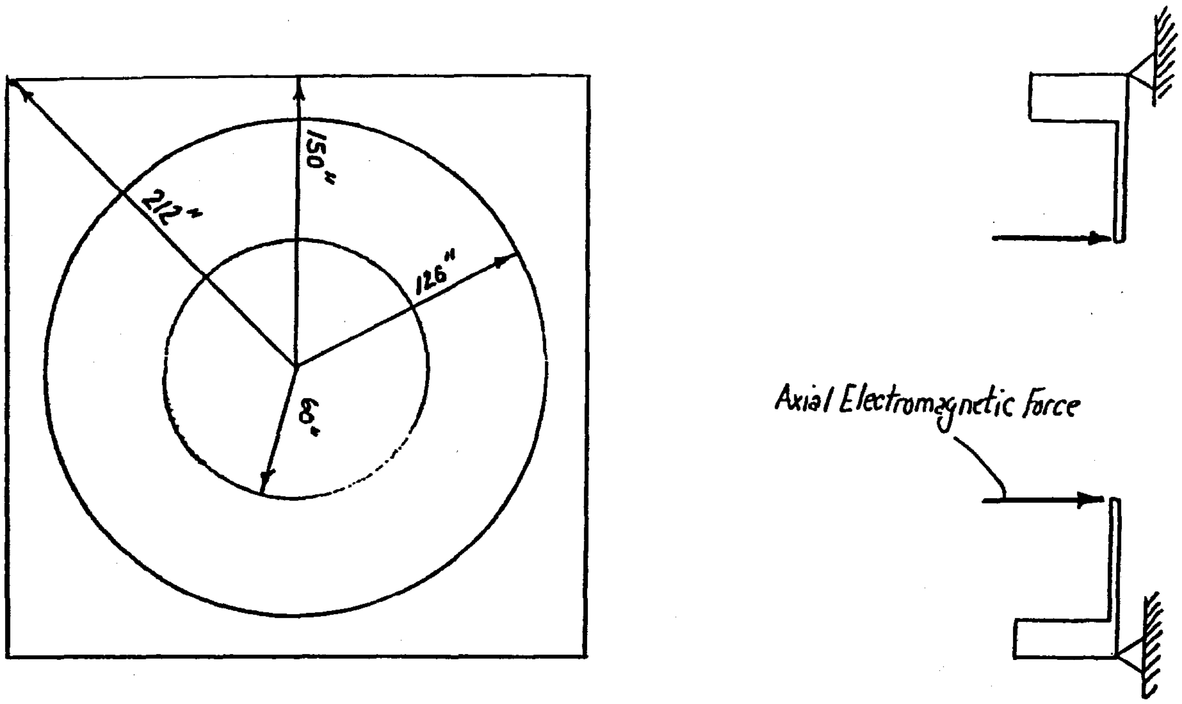


Fig 3. Endwall Analytical Model

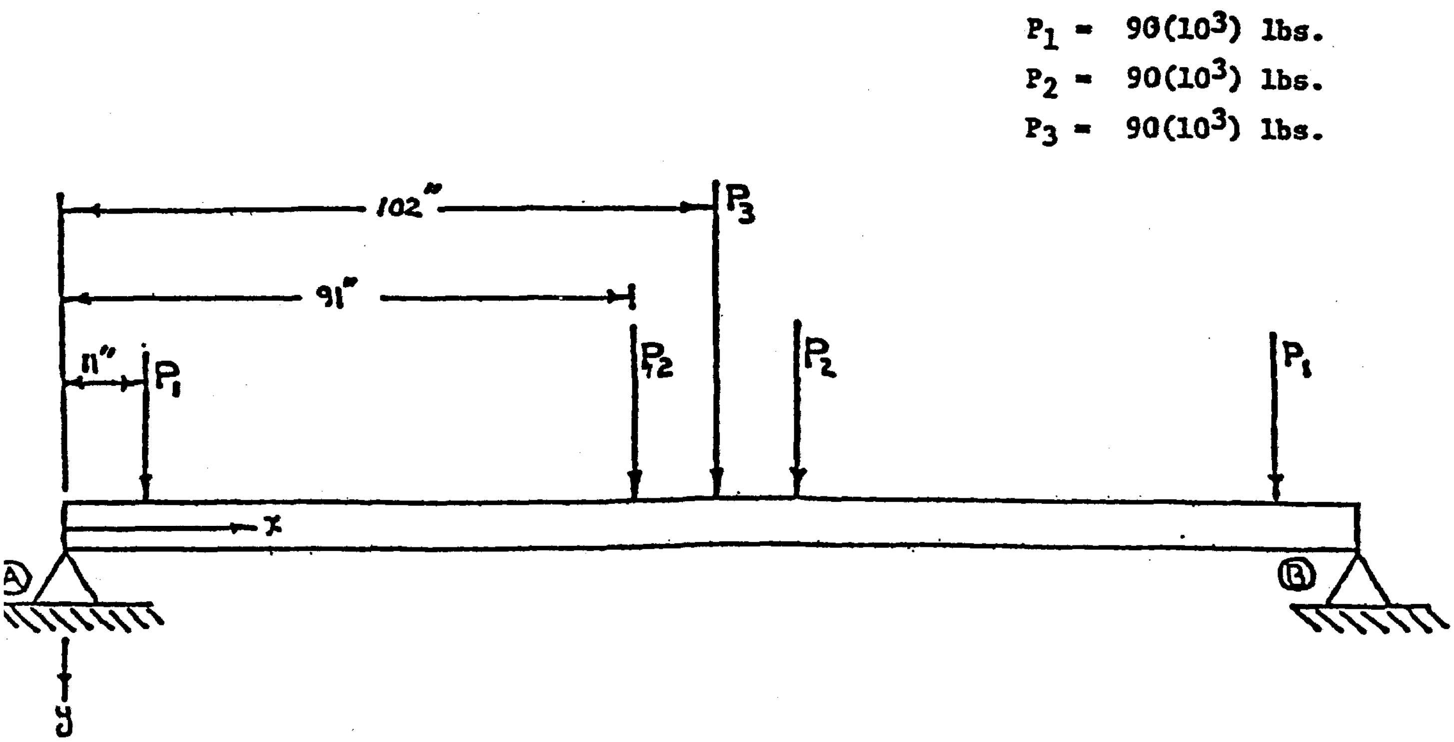


Fig 4. Lower Return Leg Analytical Model

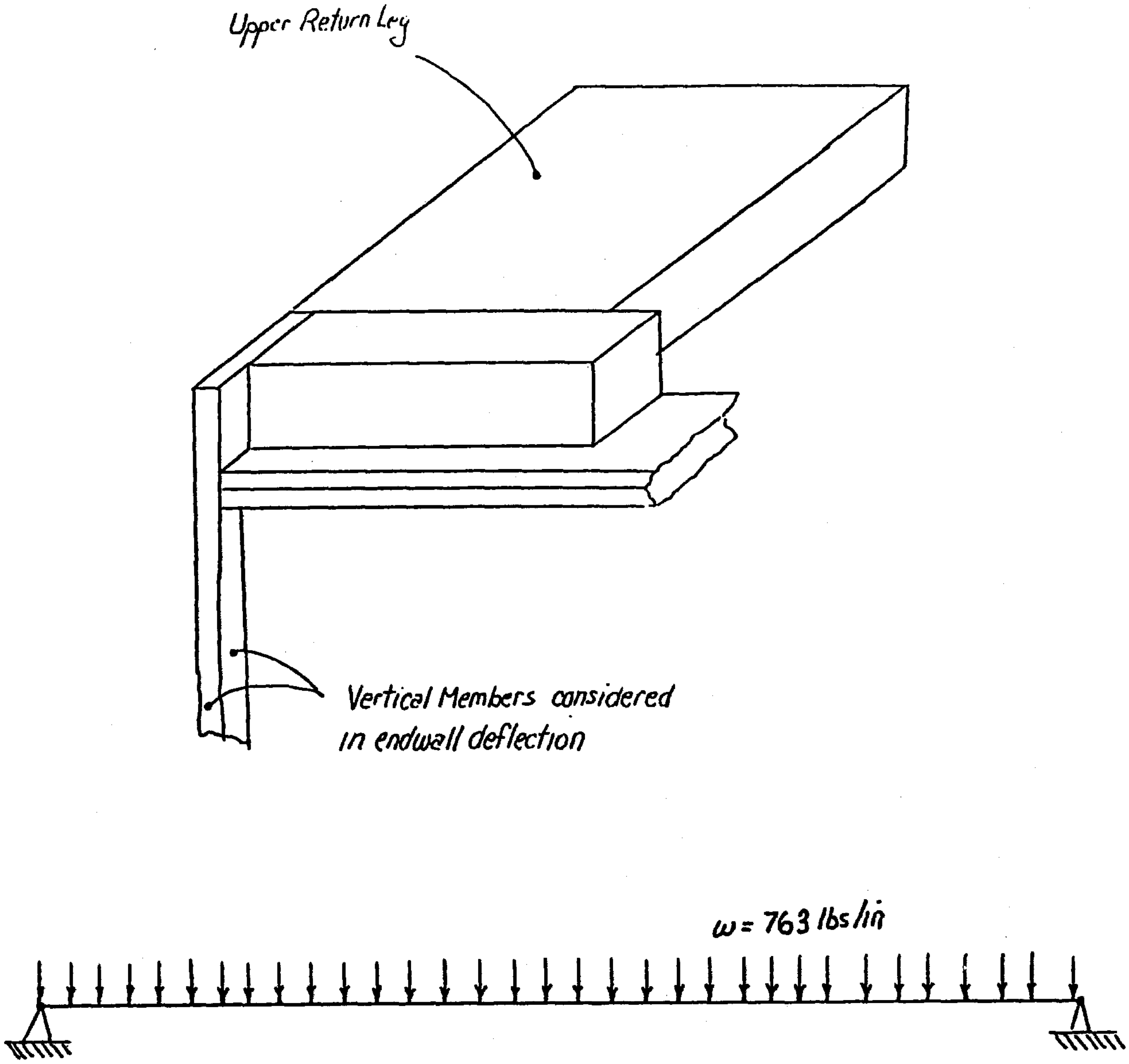
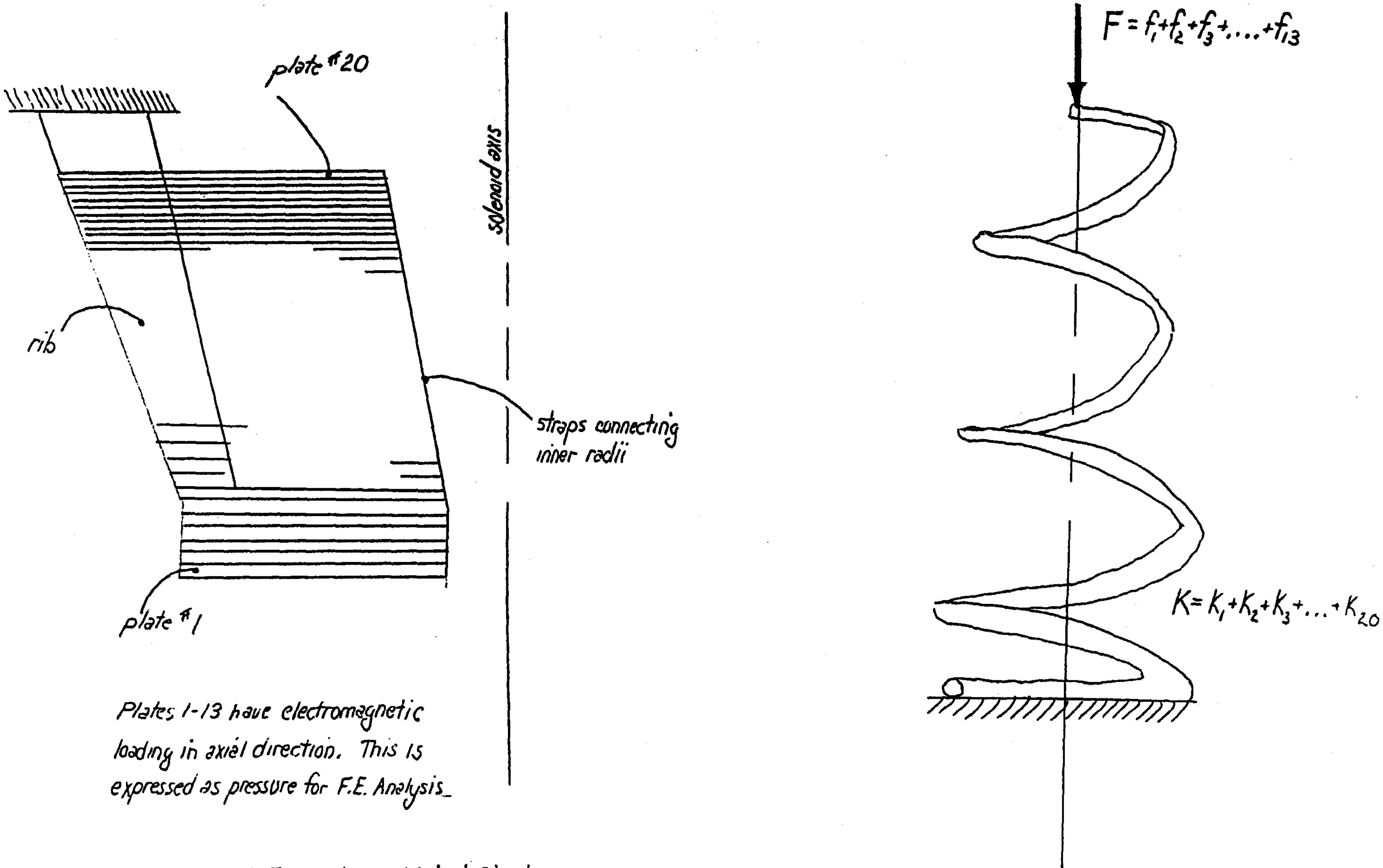


Fig 5. Upper Return Leg Analytical Model



a) F.E Model and Actual Structure

b) Analytical Model

Fig 6. Endplug Analytical Model